ISI – Bangalore Center – B Math - Physics II – End Semestral Exam
Date: 14 November 2018. Duration of Exam: 3 hours
Total marks: 75

ANSWER ALL QUESTIONS

Q1. [Total Marks: 6+4=10]

A body of constant heat capacity C_p and at temperature T_1 is put in thermal contact with a reservoir at temperature $T_2, (T_2 \neq T_1)$. Equilibrium between the body and the reservoir is established at constant pressure. Assume that C_p is temperature independent.

a.) Show that the total change of entropy is $C_p[x - 1 - \ln x]$ where $x = \frac{T_1}{T_2}$.

b.) Show that the change of entropy is positive whether $T_2 > T_1$ or $T_2 < T_1$

Q2. [Total Marks: 4+4+7+5=20]

For a general P, V, T system

a.) Derive the equation: $dS = \frac{C_v}{T} dT + \left(\frac{\partial P}{\partial T}\right)_V dV$

b.) Show that $\left(\frac{\partial C_V}{\partial V}\right)_T = T\left(\frac{\partial^2 P}{\partial T^2}\right)_V$

c.) Using the above show that the entropy for a mole of gas with the equation of state $P = \frac{RT}{V - b} - \frac{a}{V^2}$ where a and b are positive constants, is given by

$S = \int C_v(T) \frac{dT}{T} + R \ln(V - b) + const$ where C_v is a function of T.

d.) Assume that for the same system as in c.) the internal energy change under a change of temperature and volume can be written as $dU = C_v(T) dT + \frac{a}{V^2} dV$. Determine whether the temperature increases or decreases when such a gas undergoes free adiabatic expansion. Your argument needs to be rigorous and must use the above result.

Q3. [Total Marks: 4+4+2+5=15]

a.) Consider a system in thermal equilibrium with a reservoir at temperature *T*. State without derivation the probability of finding the system in a state with energy E. Your answer should include the case where there may be more than one state with the same energy.

b.) Define the partition function of such as system and derive the expression for average energy in terms of the partition function and its derivative.

c.) Consider an ideal monoatomic gas made of *N* atoms each of which has 2 internal states: a ground state and an excited state with a positive energy gap equal to Δ. The gas is in a sealed container with no energy exchange with outside world. Initially, the gas is prepared in such a way that all the atoms are in their internal ground state, but in thermal equilibrium with respect to kinetic motion of the atoms, characterized by temperature T_1 . After some time, however, due to collisions, the internal degree of freedom of the atoms is also excited and thermalized and the system temperature changes to T_2.
Give a physical explanation of why it is expected that T_2 is less than T_1.

d.) For the same system as in c.) Show that $T_2 - T_1 = -\frac{\Delta}{3k}$ upto first order in Δ. [Hint: Apply Boltzman distribution to find relative population of the two internal degrees of freedom and energy conservation]

Q4. [Total Marks: 5+5+5=15]

Consider a stretched rubber band at temperature *T*. It has length *L* when stretched with an external force *f*.

a.) Write the thermodynamic identity (1st law of thermodynamics) relating change in the internal energy *dU* due to change in length *dL* and supplied heat *TdS*.
b.) Using the fact that the free energy *U-TS* is a state function, derive the corresponding Maxwell relation $\left(\frac{\partial S}{\partial L}\right)_T = -\left(\frac{\partial f}{\partial T}\right)_L$.
c) In one experiment, the length of the band is fixed to $L = 1m$ while the temperature of the band $T = 300$K is raised by a small amount ΔT = 3K. It was found that the force needed to maintain the length of the band had to be increased by the amount $\Delta f = 1.2N$. In another experiment, the band is stretched from L to $L + \Delta L$ at constant temperature *T*. As a result the band exchanges heat with the environment. Determine the amount of heat exchanged for $\Delta L = 2$cm in Joules. Is heat released or absorbed by the band? Both experiments can be thought of as quasi-static processes. [As a check on your answer, note that the amount of heat exchanged will be one the following. 1.4 J, 2.0 J, 2.4 J.]

Q5. [Total Marks: 4+4+2+5=15]

Consider the interference pattern from three identical point slits illumined with the same coherent source of wavelength λ where the slits are apart by *d* and *3d/2* as shown in the Figure 1. (The figure is not to scale, the screen distance is much larger than *d*). Assume that these are coherent sources with amplitude *A*.

a.) Show that the intensity of light is given by
$$I = A^2\left\{3 + 2\cos\delta + a\cos\left(\frac{3\delta}{2}\right) + b\cos\left(\frac{5\delta}{2}\right)\right\}$$ where a and b are positive constants.

Determine the values of a and b . Here $\delta = \frac{2\pi d}{\lambda}\sin\theta$ is the standard phase difference from two sources *d* distance apart. Assume that θ is small and the distance to the screen is large.

b.) Show that the value of first principal maxima corresponds to when $\theta = \frac{2\lambda}{d}$. What is the value of *I* at that point?

c.) What is the value of *I* when $\theta = \frac{\lambda}{d}$?

For the last two questions, use the values of a and *b* that you have found in part a). In case you have not found these values, write your answer in terms of *a* and *b* assuming they are both positive.

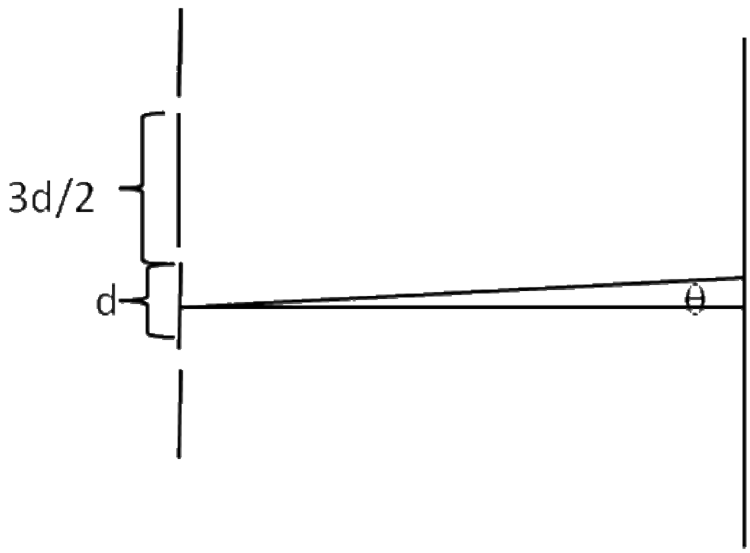
Maxwell Relations that you may use or not use:

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$$

$$\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$$

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$$

$$\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P$$


3d/2
d
θ